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Construction of the Real Numbers Via Cauchy Sequences

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Abstract—-This writing shows that construction of the real numbers via Cauchy Sequence is a mathematical concept with precise logic. The Cauchy Sequence is a concept founded by Augustine – Louis Cauchy where the sequential construction of real number is analyzed and given a theoretic frame work. The Cauchy Sequence as a function exhibit the convergence of real number as it progresses.

1 Introduction

In general, when it comes to construction of the real numbers there is a belief that they follow a successive approximation. According to (Kaye,2015) "The Cauchy property is a useful idea that describes sequences that seem to converge without mentioning any limit". For example, when we express $\pi = 4.9103$, we assume that π is a real number which has definite 4 decimal which is proportionate to the above string. In this case, it can be concluded that $|\pi - 4.9103| < 10{-}5$. However, we are not delivered with the real value of π. If, we need to arrive at an accurate approximation, we must have to calculate the decimals from 1 to 10. This means the $\pi = 4.9104568390$ and so on. However when the sequence of real numbers is not converging to a rational number then we call the process a Cauchy sequence.

2 Cauchy Sequence as Theorem

The Cauchy sequence as a theorem was instigated by Augustine – Louis Cauchy and is a mathematical concept which explains about a numerical sequence where the elements become randomly close to each other as the sequence move forward. According to (Cakalli,2015) "The concept of a Cauchy sequence involves far more than that the distance between successive terms is tending to zero". It can be said that the positive distance between the real numbers within a finite number of elements with a sequence produce lesser distance from each other at a given point. However, the Cauchy sequence in case of construction of real numbers is bounded and it would be converging in nature.

3 Cauchy Sequence as a Function

Construction of real number occurs in an ordered nature and at this system is seen as an ordered field. When the Cauchy sequence act as a function in construction of real number it is necessary to assume that real numbers are a set of rational approximating sequence with a repetitive behavior. According to (IITG,2000) "A sequence of rational numbers (a rational sequence) is a function from the natural numbers N into the rational numbers Q".

4 Construction of Real Numbers

In case of construction of real number with regard to Cauchy sequence there need to be assignment of a rational number to each real number. The Cauchy function with regard to the real number sequence is expressed by n an where the number sequence will be a1,a2,a3 and so on. According to (Watkins,2011) "A sequence {an} converges to zero if for any $\varepsilon>0$ there exists an N such that for all $n \geq N$ $|a_n|<\varepsilon$".When constructing real numbers it can be understood that as they progress they tend to converge much closer and closer and this means the Cauchy sequence is in action. Cauchy sequence with regard to sequential number is an intuition where one feels that the number is getting closer and is up to something.

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